Generalizations of Rough Sets via Topology

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Abstract

In this paper, we present a general framework for the study of rough sets using topological approaches. First, we introduce several concepts and properties of $\tau - R-$ open sets. After that, we used topology to generalize the basic rough set concepts and study their properties. Its application in data reduction and decision analysis is investigated. Finally, a simple example is adopted to demonstrate the effectiveness of the proposed models.

Keywords: Topological spaces, $\tau - R-$open sets, $\tau - R-$closed sets, rough sets

1. Introduction

Since Pawlak introduced the concept of rough set theory in 1982[20], there are many papers in which fundamental results about rough sets are considered. Rough sets have a lot of applications in various real-life fields, like information analysis, data analysis, and medicine [7, 8, 12, 13, 14, 15, 21, 22, 23, 24, 29]. In recent decades many scientists have a generalized traditional rough set theory in many ways, see [17, 18, 30, 31, 32]. Yang and Robert defined a new parameter for describing the uncertainty of rough sets in [33] and investigated roughness bounds for rough set operations in [34]. Their results demonstrate that uncertainties can be significant in a rough set. Therefore, effective representation of rough sets minimizing it's uncertainty is beneficial in rough set based data analysis. An interesting and natural way of reducing the uncertain boundary region in a rough set is studying it via topology. Topology [6] is a branch of Mathematics which is suitable for studying rough sets and information systems [2, 9, 10, 11, 25, 26, 27, 33, 35]. In this paper, we generalized the notions of rough set concepts using topological structure. New types of topological rough sets are initiated and studied new concepts of open sets. Some properties of topological rough approximations were studied. The rest of this paper is organized as follows. Section 2 presents basic concepts and facts on topological spaces and rough set approximations. Section 3 characterizes new concepts of topological open sets which are $\tau - R-$open sets and study their properties. In Section 4, we introduce new approximations

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of rough sets. Section 5 is concerned with a medical case study. And finally, some concluding remarks appear in Section 6.

2. Preliminaries

Let H be a non-empty set and τ be a topology on H, every member of τ is called an open set of H or $\tau-$open set, the complement of an open set is called a closed set of H or $\tau-$closed set and for a subset E of H, the interior and closure of E in τ are denoted by $int(E)$ and $cl(E)$, respectively. A sub collection μ of 2^H is called a supra topology on H [16] if $\emptyset$ and H belong to μ and μ is closed under arbitrary union. Let (H, μ) be a supra topological space. A subset E of H is called a supra $R-$open set if there exists a non$-$empty supra open set G such that $G \subseteq cl^{\mu}(E)$. The complement of supra $R-$open set is called supra $R-$closed set[3].

Definition 2.1.. Let (H, τ) be a topological space (TS, for short), then the subset A of H is called:

i) Regular open[28](briefly r-open) if $A = int(cl(A))$.

ii) Semi open[13](briefly s-open) if $A \subseteq cl(int(A))$.

iii) Pre- open[16] (briefly p-open) if $A \subseteq int(cl(A))$.

iv) $\gamma-$open[5] (=b-open[4]) if $A \subseteq int(cl(A)) \cup cl(int(A))$.

v) $\alpha-$open[19] if $A \subseteq int(cl(int(A)))$.

vi) $\beta-$open[1] if $A \subseteq cl(int(cl(A)))$.

Definition 2.2.. [2]Let (H, τ) be a TS and $A \subseteq H$. Then the near interior (briefly $j-$interior) of A is denoted by $int_j(E)$ for all $j \in \{r, s, p, \gamma, \alpha, \beta\}$ and is defined by $int_j(E) = \cup\{G \subseteq H : G \subseteq E, G \text{ is a } j-\text{open set}\}$.

Definition 2.3.. [2] Let $H \neq \phi, \mathbf{R}$ be any binary relation defined on H. Then $(H, \mathbf{R})$ is said to be a general approximation space, $H/\mathbf{R}$ generates a topology $\tau_{\mathbf{R}}$ and $(H, \tau_{\mathbf{R}}, \mathbf{R})$ is said to be a topological approximation space.

Definition 2.4.. [2]Let (H, τ) be a TS and $A \subseteq H$. Then the near closure (briefly j-closure) of A is denoted by $cl_j(E)$ for all $j \in \{r, s, p, \gamma, \alpha, \beta\}$ and is defined by $cl_j(E) = \cap\{F \subseteq H : E \subseteq F, F \text{ is a } j-\text{closed set}\}$.

Definition 2.5.. [2]Let $(H, \tau, \mathbf{R})$ be a topological approximation space (TAS, for short) and $A \subseteq H$. Then the near lower approximation (briefly $j-$lower approximation) of A is denoted by $\underline{R}_j(A)$ for all $j \in \{s, p, \gamma, \alpha, \beta\}$ and is defined by $\underline{R}_j(A) = int_j(A)$.

Definition 2.6.. [2]Let $(H, \tau, \mathbf{R})$ be a TAS and $A \subseteq H$. Then the near upper approximation (briefly $j-$upper approximation) of A is denoted by $\overline{R}_j(A)$ for all $j \in \{r, s, p, \gamma, \alpha, \beta\}$ and is defined by $\overline{R}_j(A) = cl_j(A)$.

Definition 2.7.. [2]Let $(H, \tau, \mathbf{R})$ be a TAS and $A \subseteq H$. Then accuracy $\alpha_j(A) = \frac{\left|\underline{R}_j(A)\right|}{\left|\overline{R}_j(A)\right|}$ for all $j \in \{r, s, p, \gamma, \alpha, \beta\}$ and $A \neq \phi$.

3. $\tau - R-$open sets

Here, we give the notion of $\tau-$R$-$open sets and studying their relations with some famous generalized $\tau-$ open sets

Definition 3.1.. Let (H, τ) be a TS. A subset E of H is called $\tau - R-$ open set if there exists $G \in \tau - \{\phi\}$ such that $G \subseteq cl(E)$. The complement of a $\tau - R-$open set is called $\tau - R-$closed set.

Remark 1.. In a TS (H, τ), the family of all $\tau - R-$open sets is denoted by $RO(H)$, $\phi \in RO(H)$ and the family of all $\tau - R-$closed sets is denoted by $RC(H)$.

Theorem 3.1.. Let (H, τ) be a TS. A subset B of H is $\tau - R-$closed iff there exists a $\tau-$closed set $F \neq H$ such that $int(B) \subseteq F$.

Proof. Suppose that B is a $\tau - R-$closed set. Now, B^c is a $\tau - R-$open set, then there exists $G \in \tau - \{\phi\}$ such that $G \subseteq cl(B^c)$. Therefore $(cl(B^c))^c \subseteq G^c$. Thus $int(B) \subseteq G^c$. Putting $F = G^c$, hence $int(B) \subseteq F \neq H$.
Conversely, consider $B \subseteq H$ and there exists a $\tau-$closed set $F \neq H$ such that $int(B) \subseteq F$, then $\phi \neq F^c \subseteq (int(B))^c = cl(B^c)$. Therefore B^c is a $\tau - R-$open set. Hence B is $\tau - R-$closed set.

Theorem 3.2.. In a TS (H, τ), every $\tau-$open set is $\tau - R-$open set.

Proof. It is clear.

Counterexample 3.1.. Let $H = \{x, y, z\}$ and $\tau = \{\emptyset, H, \{x, y\}, \{z\}\}$ be a topology on H. Then $\{y\}$ is $\tau - R-$open set, but not $\tau-$open set.

Theorem 3.3.. In a TS (H, τ) every $\tau - b-$open set is $\tau - R-$open set.

Proof. Suppose that E is a non empty $\tau - b-$open set.
Then $E \subseteq cl\ int(E) \cup int\ cl(E)) \subseteq cl(E)$. Therefore E is $\tau - R-$open set.

Counterexample 3.2.. Let $H = \{x, y, z\}$ and $\tau = \{\emptyset, H, \{x\}, \{y\}, \{x, y\}, \{y, z\}\}$ be a topology on H. Then $\{x, z\}$ is $\tau - R-$open set, but is not $\tau - b-$open set.

Proposition 3.1.. Every $\tau - R-$neighbourhood of any point in a TS (H, τ) is $\tau - R-$open set.

This proposition's proof is straight away.

Proposition 3.2.. If E is $\tau - R-$open set in a TS (H, τ), then $E \cup A$ is $\tau - R-$open set for any $A \subseteq H$.

This proposition's proof is straight away.

Theorem 3.4.. Let (H, τ) be a TS. Then the union of an arbitrary $\tau - R-$open sets is $\tau - R-$open set.

Proof. Let $\{E_i : i \in I\}$ be a family of $\tau - R-$open sets. Then there exist $i_0 \in I$ and $G \in \mu - \{\phi\}$ such that $G \subseteq cl(E_{i_0}) \subseteq cl(\cup_{i \in I} E_i)$. Hence $\cup_{i \in I} E_i$ is $\tau - R-$open set.

Remark 2.. The intersection of a finite number of $\tau - R-$open sets may not be $\tau - R-$open set as shown by the following example.

Counterexample 3.3.. Let $(\mathbb{R}, \upsilon)$ be the usual topological space on the real line. Taking $A = (3, 5], B = [5, 6)$, then A, B are $\upsilon - R-$open sets, but $A \cap B = \{5\}$ is not $\upsilon - R-$open set, because there is no $\upsilon-$open set $\phi \neq G$ such that $G \subseteq cl(\{5\})$.

Theorem 3.5.. Let (H, τ) be a TS. Then the intersection of an arbitrary $\tau - R-$closed sets is $\tau - R-$closed set.

Proof. Let $\{B_i : i \in I\}$ be a family of $\tau - R-$ closed sets. Then $\{B_i^c : i \in I\}$ is a family of $\tau - R-$open sets. Therefore $\cup_{i \in I} B_i^c$ is $\tau - R-$open set. Hence $\cap_{i \in I} B_i$ is $\tau - R-$closed set.

Definition 3.2.. Let (X, τ) be a TS, $A \subseteq X$. The $\tau - R-$interior of E (denoted by $int_R(E)$) is the union of all $\tau - R-$open sets contained in E. The $\tau - R-$closure of E (denoted by $cl_R(E)$) is the intersection of all $\tau - R-$closed sets containing E.

Theorem 3.6.. Let (H, τ) be a TS. Then

i) $A \subseteq cl_R(A)$; *and* $A = cl_R(A)$ iff A is $\tau - R-$closed set.

ii) $int_R(A) \subseteq A$; *and* $A = int_R(A)$ iff A is $\tau - R-$open set.

iii) $H - int_R(A) = cl_R(H - A)$.

iv) $H - cl_R(A) = int_R(H - A)$.

Proof. It is clear.

Theorem 3.7.. Let (H, τ) be a TS. Then

i) $int_R(A) \cup int_R(B) \subseteq int_R(A \cup B)$.

ii) $cl_R(A \cap B) \subseteq cl_R(A) \cap cl_R(B)$.

Proof. It is clear.

In the above theorem, we cannot replace inclusion relation by equality relation as shown in the following counterexamples.

Counterexample 3.4.. Let $H = \{a, b, c\}$ and $\tau = \{\emptyset, H, \{b\}, \{a, b\}, \{b, c\}\}$ be a topology on H. When $A = \{b\}$ and $B = \{c\}$, then $int_R(A) = \{b\}, int_R(B) = \emptyset$ and $int_R(A \cup B) = \{b, c\}$.

Counterexample 3.5.. Let $H = \{a, b, c\}$ and $\tau = \{\emptyset, H, \{b\}, \{a, b\}, \{b, c\}\}$ be a topology on H. When $C = \{b, c\}$ and $D = \{a, c\}$, then $cl_R(C) = H,\ cl_R(D) = \{a, c\}$ *and* $cl_R(C \cap D) = \{c\}$.

Proposition 3.3.. In a TS (H, τ) we have $\beta O(H) \subseteq RO(H)$.

Proof. Let A element in $\beta O(H)$. Then $A \subseteq cl\ int\ cl(A) \subseteq cl(A)$. Therefore A is $\tau - R-$open set. Hence $\beta O(H) \subseteq RO(H)$.

Proposition 3.4.. In a TS (H, τ) we have $\beta C(H) \subseteq RC(H)$.

Proof. Omitted.

Remark 3.. The results in this section were studied before in supra topological spaces in [5].

4. New approximations of rough sets via topology

In this section, we extended the concepts $\tau - R-$open sets and $\tau - R-$closed to rough set theory and there is a comparison between new approximations, pawlak's approximations and pre approximations. Also. we studied the properties of the new approximations.

Definition 4.1.. Let $(H, \tau, \mathbf{R})$ be a TAS, $A \subseteq H$. Then we define:

i) $\underline{R}_R(A) = int_R(A)$, $\underline{R}_R(A)$ is the lower approximation of A.

ii) $\overline{R}_R(A) = cl_R(A)$, $\overline{R}_R(A)$ is the upper approximation of A.

iii) $\alpha_R(A) = \frac{|\underline{R}_R(A)|}{|\overline{R}_R(A)|}$, $\alpha_R(A)$ is the accuracy approximation of A, where $|A|$ denotes the cardinality of $A \neq \phi$.

iv) $\mathrm{BN}_R(A) = \overline{R}_R(A) - \underline{R}_R(A)$, $\mathrm{BN}_R(A)$ is the boundary region of A.

Counterexample 4.1.. Let $X = \{a, b, c, d\}$ and $\mathbf{R} = \{(a, a), (a, c), (a, d), (b, b), (b, d), (c, a), (c, b), (c, d), (d, a)\}, \tau$ generated by $x\mathbf{R} = \{y \in U : x\mathbf{R}y\}$. Then $S = \{\{a, c, d\}, \{b, d\}, \{a, b, d\}, \{a\}\}$, therefore $\tau = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}$. Hence $\tau^C = \{X, \phi, \{b, c, d\}, \{a, c\}, \{c\}, \{b\}, \{a, b, c\}, \{b, c\}\}$.

Table 1 ($\blacksquare - R$- pen sets)

$P(X)$	cl	$\tau - R-$open or not	$\tau - R-$closed
$\{a\}$	$\{a, c\}$	yes	$\{b, c, d\}$
$\{b\}$	$\{b\}$	no	$-$
$\{c\}$	$\{c\}$	no	$-$
$\{d\}$	$\{b, c, d\}$	yes	$\{a, b, c\}$
$\{a, b\}$	$\{a, b, c\}$	yes	$\{c, d\}$
$\{a, d\}$	X	yes	$\{b, c\}$
$\{a, c\}$	$\{a, c\}$	yes	$\{b, d\}$
$\{b, c\}$	$\{b, c\}$	no	$-$
$\{b, d\}$	$\{b, c, d\}$	yes	$\{a, c\}$
$\{c, d\}$	$\{b, c, d\}$	yes	$\{a, b\}$
$\{a, b, c\}$	$\{a, b, c\}$	yes	$\{d\}$
$\{b, c, d\}$	$\{b, c, d\}$	yes	$\{a\}$
$\{a, c, d\}$	X	yes	$\{b\}$
$\{a, b, d\}$	X	yes	$\{c\}$
$\{a, b, c, d\}$	X	yes	ϕ

In table 1, we determine $\tau - R-$open subsets of X in the previous example.

The family of all $\tau - R-$open subsets of $X = \{\{a\}, \{d\}, \{a, b\}, \{a, d\}, \{a, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, X, \phi\}$.

The family of all $\tau - R-$closed subsets of $X = \{\phi, \{c\}, \{b\}, \{a\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, X\}$.

The family of all $\tau - \beta-$open subsets of $X = \{\phi, \{a\}, \{d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, X\}$.

The family of all $\tau-\beta-$closed subsets of $X = \{X, \{b,c,d\}, \{a,b,c\}, \{b,c\}, \{b,d\}, \{a,c\}, \{a, b\}, \{a\}, \{b\}, \{c\}, \phi\}$.

Table 2 (Lower, upper approximations and accuracy)

$A \subseteq X$	$\underline{R}(A)$	$\overline{R}(A)$	α	$\underline{R}_\beta(A)$	$\overline{R}_\beta(A)$	$\alpha_\beta(A)$	$\underline{R}_R(A)$	$\overline{R}_R(A)$	$\alpha_R(A)$
$\{c\}$	ϕ	$\{c\}$	0	ϕ	$\{c\}$	0	ϕ	$\{c\}$	0
$\{d\}$	$\{d\}$	$\{b,c,d\}$	$\frac{1}{3}$	$\{d\}$	$\{b,d\}$	$\frac{1}{2}$	$\{d\}$	$\{d\}$	1
$\{a,b\}$	$\{a\}$	$\{a,b,c\}$	$\frac{1}{3}$	$\{a\}$	$\{a,b\}$	$\frac{1}{2}$	$\{a,b\}$	$\{a,b\}$	1
$\{a,d\}$	$\{a,d\}$	X	$\frac{1}{2}$	$\{a,d\}$	X	$\frac{1}{2}$	$\{a,d\}$	X	$\frac{1}{2}$
$\{b,c\}$	ϕ	$\{b,c\}$	0	ϕ	$\{b,c\}$	0	ϕ	$\{b,c\}$	0
$\{c,d\}$	$\{d\}$	$\{b,c,d\}$	$\frac{1}{3}$	$\{c,d\}$	$\{b,c,d\}$	$\frac{2}{3}$	$\{c,d\}$	$\{c,d\}$	1
$\{a,b,c\}$	$\{a\}$	$\{a,b,c\}$	$\frac{1}{3}$	$\{a,c\}$	$\{a,b,c\}$	$\frac{2}{3}$	$\{a,b,c\}$	$\{a,b,c\}$	1
$\{a,b,d\}$	$\{a,b,d\}$	X	$\frac{3}{4}$	$\{a,b,d\}$	X	$\frac{3}{4}$	$\{a,b,d\}$	X	$\frac{3}{4}$

In Table 2, there is a comparison between lower approximations, upper approximations, and accuracy coefficient under consideration of Pawlak's approximations, $\beta-$open sets and $R-$open sets.

We can find the improvement of approximation accuracy by our new approximations. As we show in column 4 (which refers to accuracy in Pawlak's approximations), column 7 (which refers to accuracy in $\beta-$approximations) and column 10 (which refers to accuracy in $R-$approximations), we find that $\alpha(A) \leq \alpha_\beta(A) \leq \alpha_R(A)$ for all $A \subseteq X$.

Now, we study the properties of new approximations.

Theorem 4.1.. Let $(H, \tau, \mathbf{R})$ be a TAS. Then

1) $\underline{R}_R(A) \subseteq A \subseteq \overline{R}_R(A)$.

2) $\underline{R}_R(\phi) \subseteq \phi \subseteq \overline{R}_R(\phi)$, $\underline{R}_R(U) \subseteq U \subseteq \overline{R}_R(U)$.

3) $\overline{R}_R(X \cup Y) = \overline{R}_R(X) \cup \overline{R}_R(Y)$.

4) $\underline{R}_R(X \cap Y) \subseteq \underline{R}_R(X) \cap \underline{R}_R(Y)$.

Proof. 3) Let $x \in \overline{R}_R(X \cup Y)$. Then there exists $\tau - R-$open set G such that $x \in G$ and $G \cap (X \cup Y) \neq \phi$. Therefore $(G \cap X) \cup (G \cap Y) \neq \phi$. Thus $G \cap X \neq \phi$ or $G \cap Y \neq \phi$. So $x \in \overline{R}_R(X) \cup \overline{R}_R(Y)$. Hence $\overline{R}_R(X \cup Y) \subseteq \overline{R}_R(X) \cup \overline{R}_R(Y)$. Since $\overline{R}_R(X) \cup \overline{R}_R(Y) \subseteq \underline{R}_R(X \cup Y)$. Then $\overline{R}_R(X \cup Y) = \overline{R}_R(X) \cup \overline{R}_R(Y)$.

4) Let $x \in \underline{R}_R(X \cap Y)$. Then there exists $\tau - R-$open set G such that $x \in G$ and $G \cap (X \cap Y) \neq \phi$. Therefore $(G \cap X) \cap (G \cap Y) \neq \phi$. Thus $G \cap X \neq \phi$ and $G \cap Y \neq \phi$. So $x \in \underline{R}_R(X) \cap \underline{R}_R(Y)$. Hence $\underline{R}_R(X \cap Y) \subseteq \underline{R}_R(X) \cap \underline{R}_R(Y)$.

Remark 1. *There exists $\tau - R-$open sets which is $\tau - R-$closed.*

Examples to show that the other properties of Pawlak's approximation does not satisfied under new approximations.

Counterexample 4.2.. In Example 4.1, let $A = \{a,b,d\}$, $\underline{R}_R(A) = \{a,b,d\}$, $\overline{R}_R(\underline{R}_R(A)) = X$. Therefore $\overline{R}_R(\underline{R}_R(A)) \neq \underline{R}_R(A)$.

Counterexample 4.3.. In Example 4.1, let $A = \{b\}$, $\overline{R}_R(A) = \{b\}$, $\underline{R}_R(\overline{R}_R(A)) = \phi$. Therefore $\underline{R}_R(\overline{R}_R(A)) \neq \overline{R}_R(A)$.

Proposition 4.1.. Let $(H, \tau, \mathbf{R})$ be a TAS, $A \subseteq H$. Then $\overline{R}_R(A) \subseteq \overline{R}_\beta(A)$.

Proof. Omitted.

Proposition 4.2.. Let $(H, \tau, \mathbf{R})$ be a TAS, $A \subseteq H$. Then $BN_R(A) \subseteq BN_\beta(A)$.

Proof. Omitted.

Proposition 4.3.. In a TAS $(H, \tau, \mathbf{R})$, for $\phi \neq A \subseteq H$ we have $\alpha_\beta(A) \leq \alpha_R(A)$.

Proof. Omitted.

Proposition 4.4.. Let $(H, \tau, \mathbf{R})$ be a TAS, $A \subseteq H$. Then $\underline{R}(A) \subseteq \underline{R}_\beta(A) \subseteq \underline{R}_R(A) \subseteq A \subseteq \overline{R}_R(A) \subseteq \overline{R}_\beta(A)$.

Proof. Omitted.

5. Medical case study

In [12] Lellis et al. consider the problem of Chikungunya, a disease that is transmitted to humans by virus carrying Aedes mosquitoes. There have been recent breakouts of CHIKV associated with severe illness. It causes fever and severe joint pain. Other symptoms include muscle pain, headache, and nausea. Initial symptoms are similar to dengue fever. It is usually not life threatening. But the joint pain can last for a long time and full recovery may take months. Usually the patient gets lifelong immunity from infection and hence re-infection is very rare. In recent decades the disease has spread to Africa and Asia, in particular, the Indian subcontinent.

Consider the following information table which gives data about 8 patients and the attributes Joint pain (J), Headache (H), Nausea (N), Temperature (T), and Chikungunya.

Table 3 (Information table)

Patients	J	H	N	T	Chikungunya
P_1	Yes	Yes	Yes	High	Yes
P_2	Yes	No	No	High	No
P_3	Yes	No	No	High	Yes
P_4	No	No	No	Very High	No
P_5	No	Yes	Yes	High	No
P_6	Yes	Yes	No	Very High	Yes
P_7	Yes	Yes	No	Normal	No
P_8	Yes	Yes	No	Very High	Yes

The columns of the previous table represent the attributes (the symptoms for chikungunya) and the rows represent the objects (the patients). The entries in the table are the attribute values. The patient P_5 is characterized by the value set (Joint pain, No), (Headache, Yes), (Nausea, Yes), (Temperature, High), and (Chikungunya, No), which gives information about the patient P_5. In the table, the set of all patients is $H = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$. The attribute 'Joint pain' generates two equivalence classes, namely $\{P_1, P_2, P_3, P_6, P_7, P_8\}$ and $\{P_4, P_5\}$ where as the attributes 'Joint pain' and 'Headache' generate the equivalence classes $\{P_1, P_6, P_7, P_8\}$, $\{P_2, P_3\}$, $\{P_4\}$ and $\{P_5\}$.

The equivalence classes for the attributes Joint pain, Headache, Nausea and Temperature are $\{P_2, P_3\}$, $\{P_1\}$, $\{P_4\}$, $\{P_5\}$, $\{P_7\}$and $\{P_6, P_8\}$.

Let $X = \{P_1, P_3, P_6, P_8\}$, the set of patients having chikungunya.

By Pawlak's approximations we have, $\underline{R}(X) = \{P_1, P_6, P_8\}$, and $\overline{R}(X) = \{P_1, P_2, P_3, P_6, P_8\}$.

Observation: [12] concludes that 'Joint-pain' and 'Temperature' are the key attributes necessary to decide whether a patient has chikungunya or not. Therefore core $\{J, T\}$.

In case of the attributes 'Joint-pain':

The family of all $\tau-R-$open sets$=\{\{P_4, P_5\}\cup A, \{P_1, P_2, P_3, P_6, P_7, P_8\}\cup B : A, B \subseteq H\}$

The family of all $\tau-R-$closed sets$=\{A \subseteq H : A\cap\{P_4, P_5\} = \phi$ or $A\cap\{P_1, P_2, P_3, P_6, P_7, P_8\} = \phi\}$.

Then $\underline{R}_R(X) = \phi$, $\overline{R}_R(X) = \{P_1, P_3, P_6, P_8\}$ and $\alpha_R(X) = 0$.

In case of the attributes 'Joint pain' and 'Headache':

The family of all $\tau-R-$open sets$=\{\{P_4\}\cup A, \{P_5\}\cup A, \{P_2, P_3\}\cup A, \{P_1, P_6, P_7, P_8\}\cup A\} : A \subseteq H\}$.

The family of all $\tau - R-$closed sets$=\{A : A \subseteq H$ and $\{P_4\} \cap A = \phi$ or $\{P_5\} \cap A = \phi$ or $\{P_2, P_3\} \cap A = \phi$ or $\{P_1, P_6, P_7, P_8\} \cap A = \phi\}$. Then $\underline{R}_R(X) = \phi$, $\overline{R}_R(X) = \{P_1, P_3, P_6, P_8\}$ and $\alpha_R(X) = 0$.

In case of the attributes 'Headache', 'Nausea' and 'Temperature':

The family of all $\tau - R-$open sets $= \{\{P_1\} \cup A, \{P_4\} \cup A, \{P_5\} \cup A, \{P_7\} \cup A, \{P_2, P_3\} \cup A, \{P_6, P_8\} \cup A : A \subseteq H\}$.

The family of all $\tau - R-$closed sets $= \{A : A \subseteq H$ and $\{P_1\} \cap A = \phi$ or $\{P_4\} \cap A = \phi$ or $\{P_5\} \cap A = \phi$ or $\{P_7\} \cap A = \phi$ or $\{P_2, P_3\} \cap A = \phi$ or $\{P_6, P_8\} \cap A = \phi\}$. Then $\underline{R}_R(X) = X$, $\overline{R}_R(X) = X$ and $\alpha_R(X) = 1$.

In case of the attributes 'Headache', 'Nausea' and 'Temperature' and 'Joint pain':

The family of all $\tau-R-$open sets$=\{\{P_4\}\cup A, \{P_5\}\cup A, \{P_7\}\cup A, \{P_2, P_3\}\cup A, \{P_1, P_6, P_8\}\cup A : A \subseteq H\}$.

The family of all $\tau - R-$closed sets$=\{A : A \subseteq H$ and $\{P_4\} \cap A = \phi$ or $\{P_5\} \cap A = \phi$ or $\{P_7\} \cap A = \phi$ or $\{P_2, P_3\} \cap A = \phi$ or $\{P_1, P_6, P_8\} \cap A = \phi\}$. Then $\underline{R}_R(X) = X$, $\overline{R}^R(X) = X$ and $\alpha_R(X) = 1$.

Table 4 (Comparison of accuracy coefficients)

	$\underline{R}(X)$	$\overline{R}(X)$	α	$\underline{R}_R(X)$	$\overline{R}_R(X)$	α_R
J	ϕ	$\{P_1, P_2, P_3, P_6, P_7, P_8\}$	0	ϕ	X	0
J, H	ϕ	$\{P_1, P_2, P_3, P_6, P_7, P_8\}$	0	ϕ	X	0
H, N, T	$\{P_1, P_6, P_8\}$	$\{P_1, P_2, P_3, P_6, P_8\}$	$\frac{3}{5}$	X	X	1
J, H, N, T	$\{P_1, P_6, P_8\}$	$\{P_1, P_2, P_3, P_6, P_8\}$	$\frac{3}{5}$	X	X	1

In Table 4, there is a comparable between the accuracy in column 4 (which refers to accuracy in Pawlak's approximations) and column 7 (which refers to accuracy in our approximations) the accuracy is equal or increasing than Pawlak accuracy which shows that our new approximations are more reasonable than the traditional approximations.

6. Conclusions

In this paper, we studied general approximations of rough sets in terms of topological concepts and gave further connections between topology and rough set theory. We first characterized the $\tau - R-$ open sets and $\tau - R-$ closed sets and studied their properties and then generalized all these properties to TAS. The concepts of $\tau - R-$ boundary is very interesting and useful where the boundary region is decreased. The α_R-accuracy measure is a refinement of $\alpha_i, i \in \{S, p, \gamma, \alpha, \beta\}$.The new approximations can be applied to more general and complex information systems for future research. The topological approximation model is based on the original data only and does not need any external information, thus it is advantageous to use topological approximation spaces in most of the real-life situations.

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